

Syllabus of exam topics

American Masters in Mathematics

OMEGA

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§1 Preface

This document aims to set a syllabus for the American Masters in Mathematics. Our goal is give a straight answer to the question “what topics are considered fair game?”.

§1.1 What this syllabus is not

The syllabus is *not* the following:

- This is not an exhaustive list of theorems that participants may quote. (For diehard fans, we have a separate document called TOTAL which gives a long list of permissible theorems that we do *not* expect everyone to know.)
- This is not a binding legal contract, and is more like study guidance. That is, OMEGA does not intend for this document to chain down the problem authors. While we’ll try our best to follow the document in spirit, we reserve the right to test topics even if they don’t appear explicitly or implicitly on this list.

§1.2 Differences from other high-school math olympiads

This syllabus intentionally includes some “new” topics (particularly for 2027+). We feel that the syllabus for most olympiads is actually too narrow, and we want to encourage the best mathematical students in the country to learn a wider range of topics.

Still, our exam designers are conscious that students come from different backgrounds and we don’t expect every student to know every topic mentioned below for every problem. We make best effort so that, for example, the easiest problems do not depend too heavily on the more advanced topics in the syllabus. Our hope is that our easiest problems will be approachable for almost all students even if they have not had significant exposure to all syllabus topics.

§2 Topics in high school algebra

- Typical high school algebra topics (solving and factoring polynomials, systems of equations, inequalities, trigonometry, etc.) are required.
 - For polynomials in particular, we include relations like Vieta’s formulas and Newton sums, and the fundamental theorem of algebra.
 - Complex numbers and their basic properties (rectangular and polar form, $e^{i\theta} = \cos \theta + i \sin \theta$) are included.

The *Volume 1* and *Volume 2* textbooks published by Art of Problem Solving cover most of this ground.

- Functional equations are included. This means we expect students to understand the definition of the word “function”. Knowledge of basic properties like injectivity and surjectivity, or Cauchy’s functional equation, is also helpful.
- Basic inequalities like QM-AM-GM-HM, Cauchy-Schwarz, and Jensen inequality are included. We try to avoid the more esoteric inequalities like Schur or Muirhead that have usually been specialized specifically to competitions.
- The basic theory of sequences, series, and methods for solving linear recurrence relations is included.
- We include some basic knowledge of analysis of functions like convexity, continuity, and the intermediate value theorem. We also include the least-upper-bound and greatest-lower-bound property of $\mathbb{R}$ (that is, $\inf S$ and $\sup S$). In 2027+, this will likely be expanded to its own section (see below).

§3 Topics in combinatorics

A good reference is *Counting Rocks!* (<https://arxiv.org/abs/2108.04902>).

- Typical high-school counting methods like, permutations, $\binom{n}{k}$, principle of inclusion-exclusion, bijections, double-counting, recursion, etc., are required.
- We don’t expect knowledge of foundations of probability. Some basic knowledge of probability at a high-school level can come in handy; the linearity of expectation theorem $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$ is particularly nice to know.
- We encourage students to be familiar with terminology and the most basic results in graph theory (degrees, paths, cycles, trees, spanning trees/connectedness, bipartite graphs, etc.). We may use basic graph-theoretic terminology even in problem statements. A typical first-semester introduction to graph theory at the college level is likely to be more than sufficient.
- Although there is no particular theorems to name here, some familiarity with algorithms and processes, and techniques for analyzing them (e.g. monovariants or invariants), is expected to be helpful.
- Hall’s matching theorem and generating functions may be occasionally useful.

§4 Topics in Euclidean geometry

A good reference is *Euclidean Geometry in Mathematical Olympiads* by Evan Chen.

- Angle chasing, power of a point (radical axis), and homothety are all essential. We encourage the use of directed angles to avoid diagram dependencies.
- The use of coordinate systems (such as Cartesian coordinates, complex numbers, or barycentric coordinates) is permissible, although in many cases not necessary.
- The use of projective and inversive geometry is permissible, although in many cases not necessary.

- Trigonometry (law of sines, law of cosines, etc.) is included.
- We consider the four triangle centers (incenter I , orthocenter H , circumcenter O , centroid G) and basic facts about them (for example, the Euler line and the nine-point circle) to be standard background. We try to avoid overly obscure lemmas and configurations.
- Solid geometry is included.

§5 Topics in elementary number theory

A good reference is *Modern Olympiad Number Theory* by Aditya Khurmi.

- Familiarity with modular arithmetic is required. This includes divisibility, Fermat's theorem and Euler's generalization of it, orders, primitive roots, Chinese remainder theorem, etc.
- Quadratic residues and quadratic reciprocity (Legendre/Jacobi symbols) is included.
- GCD's and LCM's, Euclidean algorithm, Bezout lemma, etc. are included.
- For a prime p and positive integer n , considering the exponent of p in the prime factorization of n is a useful technique (this is typically denoted $\nu_p(n)$). Familiarity with ν_p techniques, or the so-called "lifting the exponent" lemma, can be helpful.
- We don't intend for number theory trivia to be emphasized. Still, at the other extreme, there are certain theorems which are famous and fundamental, and thus inevitably see practical use. We specifically include the following two theorems:

Prime number theorem The number of primes at most x is $(1 + o(1)) \frac{x}{\log x}$.

Dirichlet theorem If $\gcd(a, m) = 1$, there are infinitely many $a \bmod m$ primes.

§A Tentative topics being considered for 2027+

Here is a preview of some topics we're considering for future years of AMM. Stay tuned for further updates!

§A.1 Topics in calculus and real analysis

Broadly, calculus and real analysis at the level of a first-semester undergraduate course is permissible. We expect to be fairly similar to the Putnam in content coverage.

- This includes limits, convergence, continuity, differentiation, integration, infinite series, Rolle's theorem / mean value theorem, and Taylor expansions.
- The concept of compactness may be particularly convenient (e.g., sequential compactness, extreme value theorem, Heine-Borel, Bolzano-Weierstrass).
- For more advanced problems, knowledge of the basic definitions of point-set topology may be helpful.
- Occasionally, multivariable calculus (at a first-year college level) may be helpful. This includes partial derivatives, gradients, multiple integrals change of variables and Jacobians, Green's theorem, Divergence theorem, Stokes' theorem.

§A.2 Topics in linear algebra

We expect a typical first-semester course in linear algebra to be sufficient.

- Vector spaces and subspaces, spans, linear independence and basis, dimensions, linear maps, kernels and images, rank-nullity theorem, inner products are included.
- Matrices and determinants (and how to calculate them) are included.
- For more advanced problems, understanding of eigenvalues and eigenvectors and the spectral theorem can be helpful.

§A.3 Topics in abstract algebra

We expect a typical first-year undergraduate course in abstract algebra covering basics of groups, rings and fields would be sufficient. For some advanced problems, we may use relevant terminology even in problem statements.

Groups Groups and subgroups, cyclic groups, cosets, Lagrange theorem, normal subgroups, quotient groups, homomorphisms, group actions, orbit-stabilizer theorem.

Commutative rings Rings and ideals, polynomial rings and commutative rings, integral domains, ring homomorphisms, quotient rings, PID's, UFD's.

Fields Field extensions and their degrees, finite fields.