2026 American Masters of Mathematics
Saturday, May 23, 2026 8:30AM — 1:00PM
Denver, Colorado

Problem 1. Determine whether there exists a prime p and a positive integer N with the following property: given any N-digit positive integer whose decimal digits are all nonzero, it is possible to permute its digits to obtain a multiple of p.

Problem 2. Let $\mathbb{R}$ denote the set of real numbers. Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a strictly decreasing function such that for every real number x, the following equation holds:

$$\underbrace{f(f(\dots(f}_{2026\ f\text{'s}}(x))\dots)) = x.$$

(a) Prove that $f(f(x)) = x$ for every real number x.

(b) Prove that there exists exactly one real number x such that $f(x) = x$.

(We say a function $f\colon \mathbb{R} \to \mathbb{R}$ is *strictly decreasing* if $f(x) > f(y)$ for all real numbers $x < y$.)

Problem 3.

(a) Show that $a! + b! + 2026$ is not a perfect square for any positive integers a and b.

(b) Prove that for every positive integer n, there exists a positive integer k such that

$$a_1! + a_2! + \dots + a_n! + k$$

is not a perfect square for any positive integers $a_1, a_2, \dots, a_n$.

Problem 4. The Greek alphabet has 24 uppercase letters and 24 lowercase letters. In each cell of a 24×24 grid, we write one uppercase Greek letter and one lowercase Greek letter, with each of the 24^2 possible pairs appearing exactly once. Suppose that for each of the 48 symbols, the 24 cells containing that symbol form a connected region. Prove that the 24 cells containing the symbol Ω either all lie in the same row, or all lie in the same column.

Here, we say a set S of cells forms a *connected region* if for any two distinct cells s and s' in S, there exists a path joining them, i.e. a sequence $(s_0, s_1, \dots, s_m)$ of distinct cells in S such that $s_0 = s$, $s_m = s'$, and for each $1 \le i \le m$ the cells s_i and s_{i-1} share an edge.

Problem 5. In three-dimensional Euclidean space, nondegenerate tetrahedra $ABDE$, $BCEF$, and $CAFD$ are congruent to each other (in the same orientation, with vertices in this order). A line ℓ intersects each of the lines AD, BE, and CF. Prove that ℓ intersects the circumcircle of triangle ABC.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.